Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Partial Differential Equations/Уравнения математической физики
major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия
	Phystech School of Biological and Medical Physics
	Chair of Higher Mathematics
term:	3
qualification:	Bachelor

Semester, form of interim assessment: 6 (spring) - Grading test

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Number of course papers, tasks: 2

Authors of the program:

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The program was discussed at the Chair of Higher Mathematics 20.05.2021

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics are considered such as Harmonic functions and their properties, Cauchy problem for the wave equation.

The Cauchy problem for the heat equation, Classification of equations. Characteristics, Fourier method for solving mixed problems for the wave equation and the heat equation, Domains of external type. Boundary value problems for the Laplace equation in domains of external type, Solution of the Dirichlet problem and the Neumann problem for the Laplace equation in a circle and in a ball.

1. Study objective

Purpose of the course

The ultimate goal of the discipline "Equations of Mathematical Physics" is the formation of basic competencies together with the underlying knowledge, skills and abilities of using a standard mathematical apparatus designed to describe physical processes that depend on two or more variables. Typically, such processes are described by partial differential equations. And although in the most interesting cases the equations turn out to be nonlinear, the simplest way to construct a theory of even nonlinear partial differential equations of the second and higher order begins with the linearization of such equations. Due to the fact that the introduction to the theory of quasilinear partial differential equations of the first order was included in the previous course of ordinary differential equations, the general goal of the introductory course in the basic mathematical apparatus for describing multidimensional physical processes is traditionally reduced to the study of methods for solving correctly posed problems of mathematical physics, formulated as problems with initial, boundary and initial-boundary conditions for linear second-order partial differential equations. In this case, equations of order higher than the second, as a rule, remain outside the standard introductory course, despite their importance, for example, for mechanics of continuous media and the theory of elasticity. The main goal of this introductory course is to master the basic classical approaches to solving correctly posed problems, using both analytical methods of solution, supplemented with elements of modern methods, and qualitative methods of analyzing the sought solutions, applicable even when the analytical form of the solutions themselves is not known. Concrete classical problems solved in the course by classical methods should not be perceived purely utilitarian, as solutions of certain problems that can be applied to something, but cannot be applied directly to something. The fundamental motivation for this course should be considered an introduction to classical approaches to classical problems of mathematical physics, which should be perceived rather as the simplest and most understandable samples and examples that can and should be guided by a researcher who poses and solves urgent problems of modern mathematical physics.

Tasks of the course

To master all stages of solving the problem of mathematical physics according to the complete scheme:

“Classification of the problem - analysis of the correctness of the formulation - the choice of a suitable analytical method for the solution - solution of the problem - analysis of the found solution”. Also master all the stages of the analysis of a problem of mathematical physics of a general type according to an incomplete scheme:

"Classification of the problem - analysis of the correctness of the statement - qualitative analysis of the properties of the desired solution" in the case when the problem does not lend itself to an analytical solution in an explicit form, which for partial differential equations is more a general rule than an exception. In practice, such an analysis makes it possible to quickly determine the correct direction of the search for any other means of solving the problem, in addition to analytical ones, such as, for example, approximate and numerical methods, although based on the MFM course, but going beyond its traditional framework.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them

UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- the main types of partial differential equations;
- determination of the characteristic surface;
- basic boundary value problems for equations of hyperbolic type, parabolic type, elliptic type;
- d'Alembert, Poisson, Kirchhoff formulas for the solution of the Cauchy problem for the wave equation;
- maximum principles for parabolic and elliptic equations;
- Fourier method for constructing classical solutions of initial-boundary value problems for the heat equation and the wave equation;
- basic properties of harmonic functions;
- poisson's formula for the solution of the Dirichlet problem for the Laplace equation in a ball;
- poisson's formula for the solution of the Neumann problem for the Laplace equation in a ball.

be able to:

- determine the type of partial differential equations; reduce second order equations with variable coefficients to the canonical form;
- to solve by the method of characteristics of the Cauchy and Goursat problem for the hyperbolic equation on the plane;
- to solve mixed problems on the semiaxis for a one-dimensional wave equation;
- solve the Cauchy problem for the wave equation;
- solve the Cauchy problem for the heat equation;
- to apply the Fourier method when solving mixed problems for the wave equation and the heat equation;
- to solve boundary value problems for the Poisson equation in circular and spherical regions.

master:

- determine the type of partial differential equations; reduce second order equations with variable coefficients to the canonical form;
- to solve by the method of characteristics of the Cauchy and Goursat problem for the hyperbolic equation on the plane;
- to solve mixed problems on the semiaxis for a one-dimensional wave equation;
- solve the Cauchy problem for the wave equation;
- solve the Cauchy problem for the heat equation;
- to apply the Fourier method when solving mixed problems for the wave equation and the heat equation;
- to solve boundary value problems for the Poisson equation in circular and spherical regions.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory	Independent

		Lectures	Seminars	practical	work
1	Harmonic functions and their properties.	4	4		10
2	Cauchy problem for the wave equation.	5	5		12
3	Cauchy problem for the heat equation.	4	4		10
4	Classification of equations. Characteristics.	4	4		11
5	The Fourier method for solving mixed problems for the wave equation and the heat equation.	5	5		12
6	Areas of the outer type. Boundary value problems for the Laplace equation in domains of external type.	4	4		10
7	Solution of the Dirichlet problem and the Neumann problem for the Laplace equation in a circle and in a ball.	4	4		10
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 6 (Spring)

1. Harmonic functions and their properties.

Harmonic functions. Fundamental solution to the Laplace equation. Potentials of single and double layers. Volume (Newtons) potential. Infinite differentiability of harmonic functions. Average theorems. Singularity elimination theorem. Maximum principle. Liouville's theorem.

2. Cauchy problem for the wave equation.

Wave equation in the case of two and three spatial variables. Flat characteristics of the wave equation, light cone. Statement of the Cauchy problem. Cauchy problem for the wave equation. Necessary conditions for the existence of a solution. Energy conservation law and uniqueness of the solution to the Cauchy problem. Existence of a solution to the Cauchy problem in the cases of three spatial variables (Kirchhoff's formula). Existence of a solution to the Cauchy problem in the case of two spatial variables (Poisson's formula, descent method). Continuous dependence of the solution on the initial functions.

Wave propagation in the case of two and three spatial variables. Diffusion of waves in the case of two spatial variables.

3. Cauchy problem for the heat equation.

The Cauchy problem for the heat equation. Necessary conditions for the existence of a solution. Fundamental solution of the heat conduction equation. Uniqueness of the solution limited in each characteristic strip. Tikhonov's uniqueness class. Solution of the Cauchy problem for the homogeneous heat equation-Poisson's formula. Infinite differentiability of the solution. Maximum principle. Continuous dependence of the solution on the initial function. The absence of a continuous dependence of the solution of the Cauchy problem for the equation of "inverse heat conduction" (example of Hadamard).

4. Classification of equations. Characteristics.

Partial differential equations. Linear differential equations. Classification of second-order equations.

Characteristics of second-order linear equations. Ordinary differential equation for characteristics in the two-dimensional case. Characteristics of the wave equation.

Wave equation in the case of one spatial variable. Statement of the Cauchy problem (in particular, the localized problem), d'Alembert formula. Dependence domain of the solution to the Cauchy problem. Continuous dependence of the solution on the initial functions. An example of the absence of a continuous dependence in the case of the Laplace equation (Hadamard's example).

5. The Fourier method for solving mixed problems for the wave equation and the heat equation.

Mixed problem for a one-dimensional heat equation on a finite segment. Necessary conditions for the solvability of the problem (smoothness conditions for the right-hand side of the equation and the initial and boundary functions and the conditions for their agreement). The maximum principle and the uniqueness theorem. A theorem on the continuous dependence of the solution on the initial and boundary functions.

Fourier's method of proving the theorem on the existence of a solution.

Mixed problem for a one-dimensional wave equation on a finite segment. Necessary conditions for the solvability of the problem (smoothness conditions for the right-hand side of the equation and the initial and boundary functions and the conditions for their agreement). The uniqueness theorem and the theorem on the continuous dependence of the solution on the initial functions (energy conservation law).

Fourier's method of proving the theorem on the existence of a solution.

6. Areas of the outer type. Boundary value problems for the Laplace equation in domains of external type.

Uniqueness of the solution to the external Dirichlet problem. Uniqueness of the solution to the external Neumann problem.

7. Solution of the Dirichlet problem and the Neumann problem for the Laplace equation in a circle and in a ball.

The Dirichlet problem for the Poisson equation in a bounded domain. Necessary conditions for its solvability. Uniqueness of the solution; continuous dependence of the solution on the boundary function. Solution of the Dirichlet problem for the Laplace equation in the ball-Poisson formula.

Neumann's problem for the Poisson equation in a bounded domain. Necessary conditions for solvability. Theorem on the general form of the solution to the problem. Solution of the Neumann problem for the Laplace equation in a ball.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Уравнения с частными производными [Текст] = Partial differential equations/Л. Берс, Ф. Джон, М. Шехтер , -М., Мир, 1966

Additional literature

1. Лекции об уравнениях математической физики [Текст] : [учеб. пособие для вузов] / М. А. Шубин .— 2-е изд., испр. — М. : МЦНМО, 2003 .— 303 с.

7. List of web resources that are necessary for the course (training module) mastering

1. http://math.stackexchange.com – international educational mathematical website, which presents almost all branches of mathematics, including pde (=partial differential equations); the best available on the Internet means of free mathematical self-education; it works successfully thanks to the participation of students and teachers of universities around the world: students are looking for and find help in solving problems, and teachers have access to a huge database of problems to any standard mathematical course in the University program from baklavriata to graduate school, with fresh problems come to the site around the clock continuous flow; guidelines for effective use of the site, see below in section 10;
2. http://eqworld.ipmnet.ru/ – source of information on linear and nonlinear differential and functional equations with online access to textbooks and reference books on mathematics, mechanics and physics and with links to other similar Internet sources;
3. http://www.wolframalpha.com/ – a new way to gain knowledge and get answers to questions: not by searching on the Internet, but by accessing online the use of extensive databases, algorithms and methods;
4. http://www.encyclopediaofmath.org – Mathematical encyclopedia, published in Moscow in Russian in 5 volumes by the publishing house "Soviet encyclopedia" in 1977, and then reprinted in English by Kluwer Academic Publishers in 2002 with the comments of experts.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

9. Guidelines for students to master the course

Successful mastering of the course requires intense independent work of the student. The program of the course indicates the time for independent work, the minimum necessary for a full-fledged student's work on the topic. Independent work includes:
- reading and taking notes of the recommended literature,
- study of educational material (based on lecture notes, educational and scientific literature), preparation of answers to questions intended for independent study, proof of individual statements, properties;
- solving problems offered to students in lectures and practical classes,
- preparation for practical training.

Guidance and control over the student's independent work is carried out in the form of individual consultations.

An indicator of mastery of the material is the ability to solve problems. To form the ability to apply theoretical knowledge in practice, the student needs to solve as many problems as possible. When solving problems, each action must be adequately argued, referring to already assimilated theoretical information. A well-executed drawing that reflects the conditions of the problem will help to facilitate the solution of the problem.

In preparation for practical exercises, it is necessary to repeat the previously studied basic definitions, theorem formulations. Usually they adhere to the following scheme: study of the material of the lecture on the synopsis on the same day when the lecture was listened to (10-15 minutes); repetition of the material on the eve of the next lecture (10-15 minutes), study of educational material based on lecture notes, educational and scientific literature, preparation of answers to questions intended for self-study (1 hour a week), preparation for a practical lesson, problem solving (1 hour) ... It is important to achieve an understanding of the material being studied, not its mechanical memorization. If you find it difficult to study certain topics, issues, you should seek advice from a lecturer or teacher leading practical classes.

SUPPLEMENT

Assessment funds for course (training module)

major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия Phystech School of Biological and Medical Physics Chair of Higher Mathematics
term:	3
qualification:	Bachelor

Semester, form of interim assessment: 6 (spring) - Grading test

Authors:

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T.V. Mikhaylova, candidate of physics and mathematical sciences, associate professor, associate professor
O.G. Podlipskaya, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- the main types of partial differential equations;
- determination of the characteristic surface;
- basic boundary value problems for equations of hyperbolic type, parabolic type, elliptic type;
- d'Alembert, Poisson, Kirchhoff formulas for the solution of the Cauchy problem for the wave equation;
- maximum principles for parabolic and elliptic equations;
- Fourier method for constructing classical solutions of initial-boundary value problems for the heat equation and the wave equation;
- basic properties of harmonic functions;
- poisson's formula for the solution of the Dirichlet problem for the Laplace equation in a ball;
- poisson's formula for the solution of the Neumann problem for the Laplace equation in a ball.

be able to:

- determine the type of partial differential equations; reduce second order equations with variable coefficients to the canonical form;
- to solve by the method of characteristics of the Cauchy and Goursat problem for the hyperbolic equation on the plane;
- to solve mixed problems on the semiaxis for a one-dimensional wave equation;
- solve the Cauchy problem for the wave equation;
- solve the Cauchy problem for the heat equation;
- to apply the Fourier method when solving mixed problems for the wave equation and the heat equation;
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master:

- determine the type of partial differential equations; reduce second order equations with variable coefficients to the canonical form;
- to solve by the method of characteristics of the Cauchy and Goursat problem for the hyperbolic equation on the plane;
- to solve mixed problems on the semiaxis for a one-dimensional wave equation;
- solve the Cauchy problem for the wave equation;
- solve the Cauchy problem for the heat equation;
- to apply the Fourier method when solving mixed problems for the wave equation and the heat equation;
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3. List of typical control tasks used to evaluate knowledge and skills

The current control is carried out on the basis of the fulfillment by students of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current performance are entered by teachers in special journals.

Current control based on homework is carried out during the academic semester within the timeframe established by the Educational Department, in accordance with the curriculum.

To pass the assignment, the student must provide a solution to the homework problem in writing, answer the teacher's questions and write a test on the assignment, which tests the knowledge of concepts and statements on the topics of the assignment being handed over and the ability to solve problems.

4. Evaluation criteria

Certification in the discipline "Partial Differential Equations/Уравнения математической физики" is carried out in the form of a differential test.

Differentiated test is carried out orally, taking into account the control tasks previously completed by students.

Control tasks:

1. Linear equation of the second order with constant coefficients. Classification.
2. Reduction to the canonical form.
3. Equations of Laplace, Poisson, wave equation, heat conduction and others.
4. General solutions. Transformations that preserve the form of the equation. The principle of superposition of solutions.
5. Duplication of solutions. Self-similar solutions. Examples.
6. Linear equation of the second order with variable coefficients.
7. Change of independent variables and reduction to the canonical form of an equation with two independent variables.
8. Classification in point and in area. Characteristics.
9. Statement of problems in mathematical physics. Typical tasks. Boundary and initial conditions.
10. Tasks: Cauchy; edge; mixed.
11. Multidimensional shift operators. Properties. Applications.
12. The Cauchy problem and the representation of its solution for a linear first-order partial differential equation.
13. Solution of the Cauchy problem (wave equation; heat conduction equation) for quasi-polynomial input data.
14. Cauchy problem for the equation of string vibrations (n = 1). D'Alembert formula.
15. Problems for a semi-bounded line.
16. The Cauchy problem for the wave equation (n = 3). Kirchhoff's formula.
17. Cauchy problem for the equation of membrane vibrations (n = 2). Poisson's formula (descent method). Uniqueness of the solution to the Cauchy problem.
18. Linear hyperbolic high-order partial differential equation with two independent variables. Cauchy problem and presentation of its solution. Characteristics.
19. Linear hyperbolic system of partial differential equations of the first order and with two independent variables. Cauchy problem and presentation of its solution. Characteristics.
20. Mixed problem for the equation of vibrations of a string on a segment. Uniqueness of the solution.
21. The principle of superposition of solutions and the method of separation of variables.
22. The problem with data on the characteristic (Goursat problem).
23. Sturm-Liouville problem on a segment. Green's function of the Sturm - Liouville operator.
24. Equations of Laplace, Poisson and Helmholtz. Harmonic functions and their properties.
25. The principle of maximum. Kelvin transform.
26. Laplacian in polar, cylindrical and spherical coordinates.
27. Fundamental solutions (n = 1, n = 2, n = 2, n \ geq4) and delta function.
28. Basic boundary value problems (internal; external): Dirichlet problem; Neumann's problem.
29. Theorems of uniqueness. Green's formulas. Green's function of the Dirichlet problem.

30. Representation of the solution of the Dirichlet problem (in a circle and a ball, in a half-plane and a half-space) in terms of boundary values.
31. Conformal mappings and their use for solving the Dirichlet problem and for constructing the Green's function.
32. The method of separation of variables and the solution of boundary value problems in R ^ 2.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

When conducting differential credit, the student is given 1 astronomical hour for preparation. The student's questionnaire on the test should not exceed two astronomical hours.
During the differential offset, students can use only the discipline program.